Extensive Robustness and Complexity of Entanglement in Quantum Spin Chains

Jovan Odavić, G. Torre, N. Mijić, D. Davidović, F. Franchini, S. M. Giampaolo
Ruđer Bošković Institute - Zagreb (Croatia)
Integrable and Chaotic Dynamics Workshop, Pokljuka (Slovenia) 2022

Institut Ruđer Bošković

Abstract

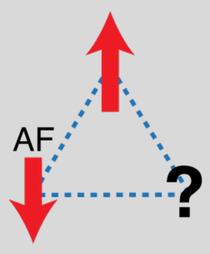
Entanglement is recognized as a primary resource for quantum advantage. We explore the resilience of various kinds of entanglement in the ground state of a local Hamiltonian against the application of random local unitaries, acting at most on two neighboring sites of a 1D spin chain. We distinguish two classes of operations, depending on whether they preserve or not the symmetries of the Hamiltonian. The latter are more efficient in destroying the entanglement, but also change its nature by making it more complex. Adding topological frustration to the chain adds additional, non-local entanglement, which cannot be completely destroyed by the local unitaries. Our work highlights a subtle interplay between locality and non-local constraint.

Model and Physics – Transverse-field Ising model (TFIM)

- We consider a minimal model with non-extensive geometrical frustration

$$H = J\sum_{j=1}^{N}\sigma_j^x\sigma_{j+1}^x - h\sum_{j=1}^{N}\sigma_j^z \tag{1}$$

  - assume PBC $\sigma_{N+1}^x = \sigma_1^x$ with and odd number of spins $N = 2M + 1$ $(M \in \mathbb{Z})$, with antiferromagnetic coupling $\rightarrow$ Frustrated Boundary Conditions (FBC) [1]
- Classical non-extensive geometric frustration



  - $2N$ degenerate ground-state manifold at $h = 0$, while in quantum regime $h \neq 0$ lifted degeneracy and the TD limit $N \rightarrow \infty$ gapless spectrum
- To quantify the entanglement we use the bipartite Rényi-2 entropies

$$S(\rho_A) = -\log \mathrm{Tr}\left[\rho_A^2\right], \tag{2}$$

  - where $\rho_A = \mathrm{Tr}_{A^c}|\psi\rangle\langle\psi|$ is the reduced density matrix, $\{\lambda_i\}_{i=1}^{2^{N/2}}$ we denote the ordered set (decreasing order) of eigenvalues.

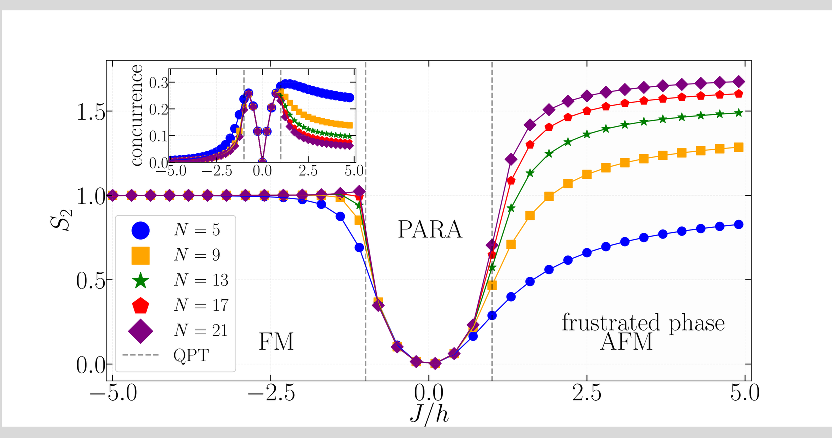


Figure: Half-chain Rényi-2 entropy for increasing system sizes. Inset: Averaged nearest-neighbor concurrence.

Method - Entanglement cooling stochastic circuit

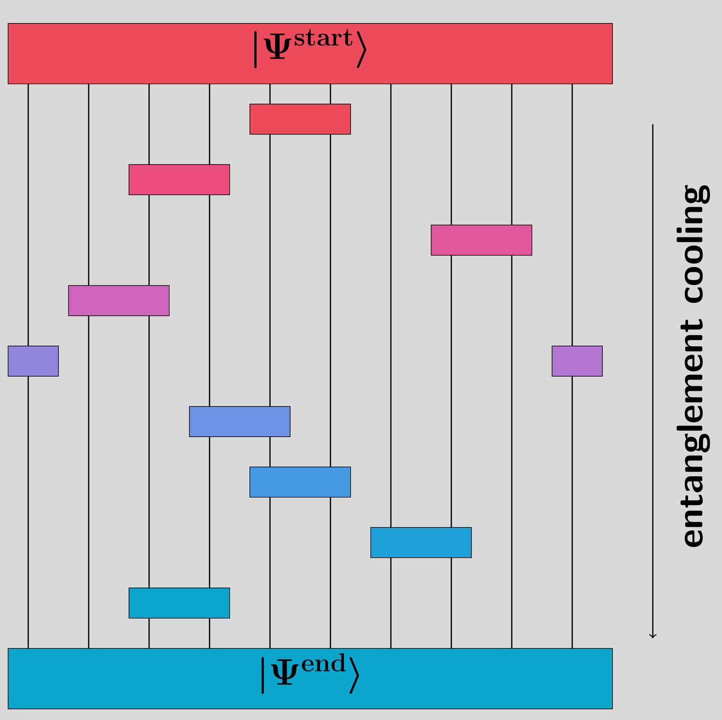


Figure: Cartoon of the entanglement cooling algorithm in terms of a quantum circuit. Provided with a quantum many-body state $|\Psi^{\text{start}}\rangle$ random local unitaries act at most on two neighboring sites with periodic boundary conditions assumed. The acceptance probability is implemented as a temperature gradient. The local gates can be written as $U_k = \exp\left(ih_j^{(k)}\Delta t\right)$ with $\Delta t = \pi/10$ and are applied at randomly selected neighboring pairs of spins $\{j, j+1\}$. Simulated annealing Metropolis Monte Carlo algorithm that targets to disentangle the TFIM ground-state $\rightarrow$ if we can not destroy entanglement we say the state is ROBUST (STOCHASTIC IRREVERSIBILITY)! [2]

Gate set 1	Gate set 2
$h_j^{(1)} = \sigma_j^z \otimes \mathbb{I}_{j+1} + \mathbb{I}_j \otimes \sigma_{j+1}^z$	$h_j^{(4)} = \sigma_j^x \otimes \mathbb{I}_{j+1} + \mathbb{I}_j \otimes \sigma_{j+1}^x$
$h_j^{(2)} = \sigma_j^x \otimes \sigma_{j+1}^x$	$h_j^{(5)} = \sigma_j^y \otimes \mathbb{I}_{j+1} + \mathbb{I}_j \otimes \sigma_{j+1}^y$
$h_j^{(3)} = \sigma_j^y \otimes \sigma_{j+1}^y$	$h_j^{(6)} = \sigma_j^z \otimes \sigma_{j+1}^z$

Table: Table of local deterministic operators. Gates set 1 is **non-universal** and parity preserving. Gate sets 1 & 2 taken together form the **universal** set.

Results 1 – Disentanglement slow-down, extensive irreversibility

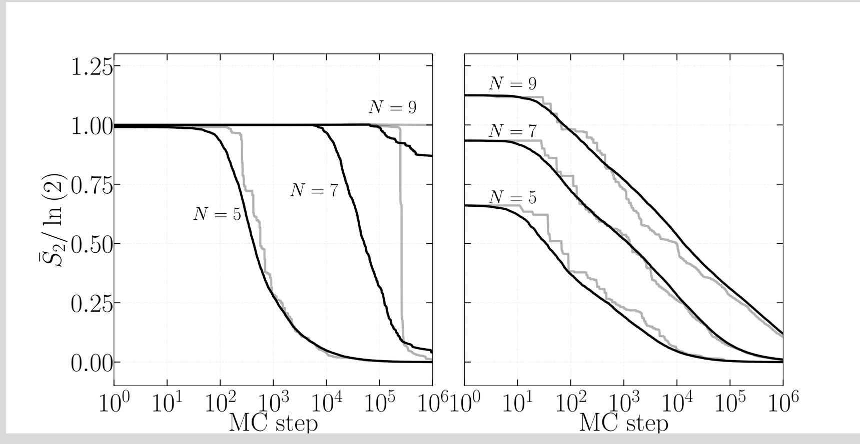


Figure: Averaged rescaled Rényi-2 entropy during the entanglement cooling over $M = 10^2$ Metropolis MC trajectories. Left: unfrustrated system $J/h = -2.5$ in the ferromagnetic (FM) regime; Right: frustrated system $J/h = 2.5$ in the antiferromagnetic (AFM) phase. Superimposed (silver line) we see a single MC trajectory for each of the system sizes. Universal set of gates used.

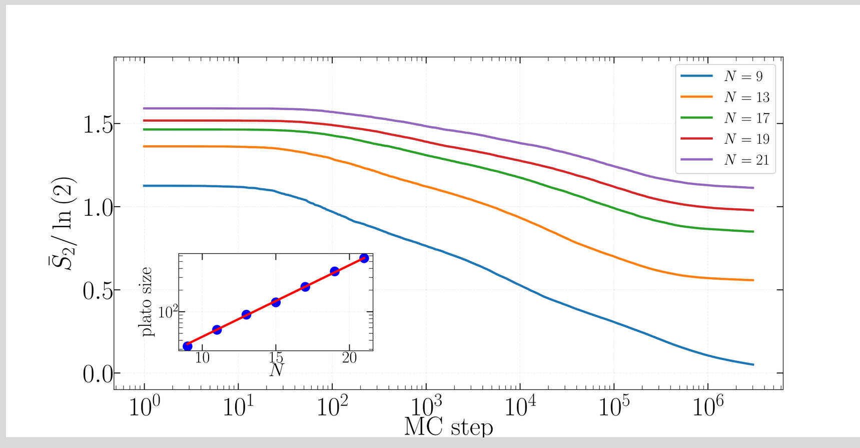


Figure: Averaged rescaled Rényi-2 entropy during the entanglement cooling over $M = 10^2$ Metropolis MC trajectories with different system size in the frustrated regime $(J/h = 2.5)$. Inset shows the scaling of the plato size of the entanglement during the cooling (blue dots). The plato is an exponential function of the system size. In red is an exponential function fit. Universal set of gates used.

Results 2 – Entanglement spectrum, consecutive spacing ratio $P(r)$

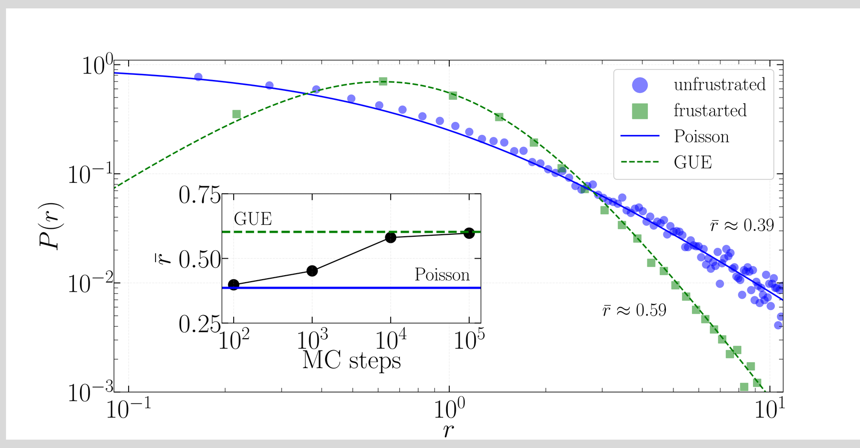


Figure: Entanglement spectrum eigenvalue differences of the unfrustrated $(J/h = -2.5)$ vs frustrated $(J/h = 2.5)$ state for the system size $N = 17$ after the application of the cooling algorithm for 10^5 steps with $M = 10^2$ realizations. Th spacing is defined $r_i = (\lambda_{i+1} - \lambda_i)/(\lambda_i - \lambda_{i-1})$. The predictions from RMT are given with lines. Inset show the average consequetive spacing ratio with variable Metropolis MC steps for the frustrated system of size $N = 17$. The Random Matrix Theory (RMT) provides predictions $\bar{r}_{\text{Poisson}} = 2\ln 2 - 1 \approx 0.386$ and $\bar{r}_{\text{WD}} = 2\sqrt{3}/\pi - 1/2 \approx 0.602$. Universal set of gates used.

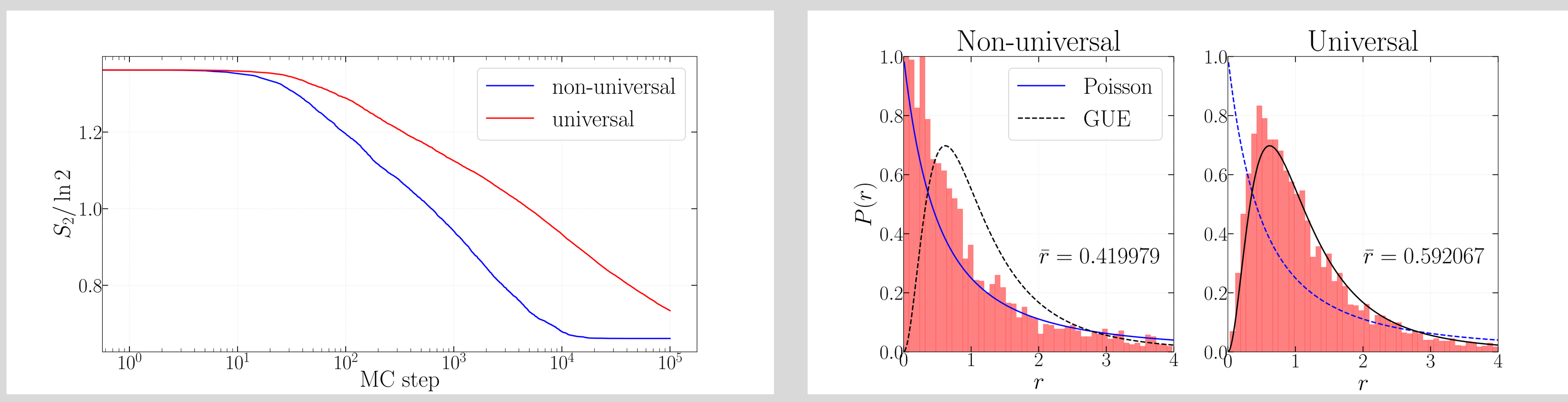


Figure: Entanglement cooling performed with different set of unitary gates. *Left panel*: Averaged rescaled Rényi-2 entropy for $N = 13, M = 10^2$ and $J/h = 2.5$. Clear difference is observed. *Right panel*: Consecutive spacing ratio of the entanglement spectrum for the two sets of gates (non-universal vs universal).

Conclusion

- Stark difference between unfrustrated vs frustrated regime, and extensive slow-down of entanglement destruction!
- Entanglement spectrum complexity signature between non-universal and universal!
- Used Graphical Processing Units (GPU) to perform efficient calculations [3]

This work is supported by the Croatian Science Foundation under the grant HRZZ-UIP-2020-02-4559 and HRZZ-IP-2019-4-3321.

HRZZ Croatian Science Foundation
Hybrid Scale
SCAN ME

References

[1] S. M. Giampaolo, F. B. Ramos, and F. Franchini, Journal of Physics Communications **3**, 081001 (2019), ISSN 2399-6528, URL https://doi.org/10.1088/2399-6528/ab3ab3.
[2] C. Chamon, A. Hamma, and E. R. Mucciolo, Physical Review Letters **112**, 240501 (2014), URL https://link.aps.org/doi/10.1103/PhysRevLett.112.240501.
[3] N. Mijić and D. Davidović, arXiv:2203.09353 [cs] (2022), URL http://arxiv.org/abs/2203.09353.