

# Experimental demonstration of non-local magic in a superconducting quantum processor

\*or

The story of bringing experiment and theory together  
(Pragma (πρᾶγμα): Enduring, long-lasting love that develops over a long time.)



Theory



Experiment

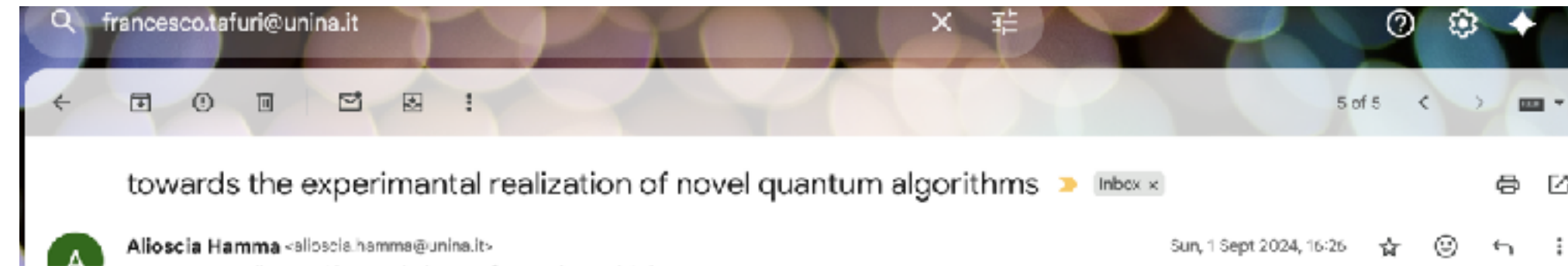
# UniNA quantum computer

- Unina host the first public quantum computer in Italy
- We were fortunate to start a collaboration

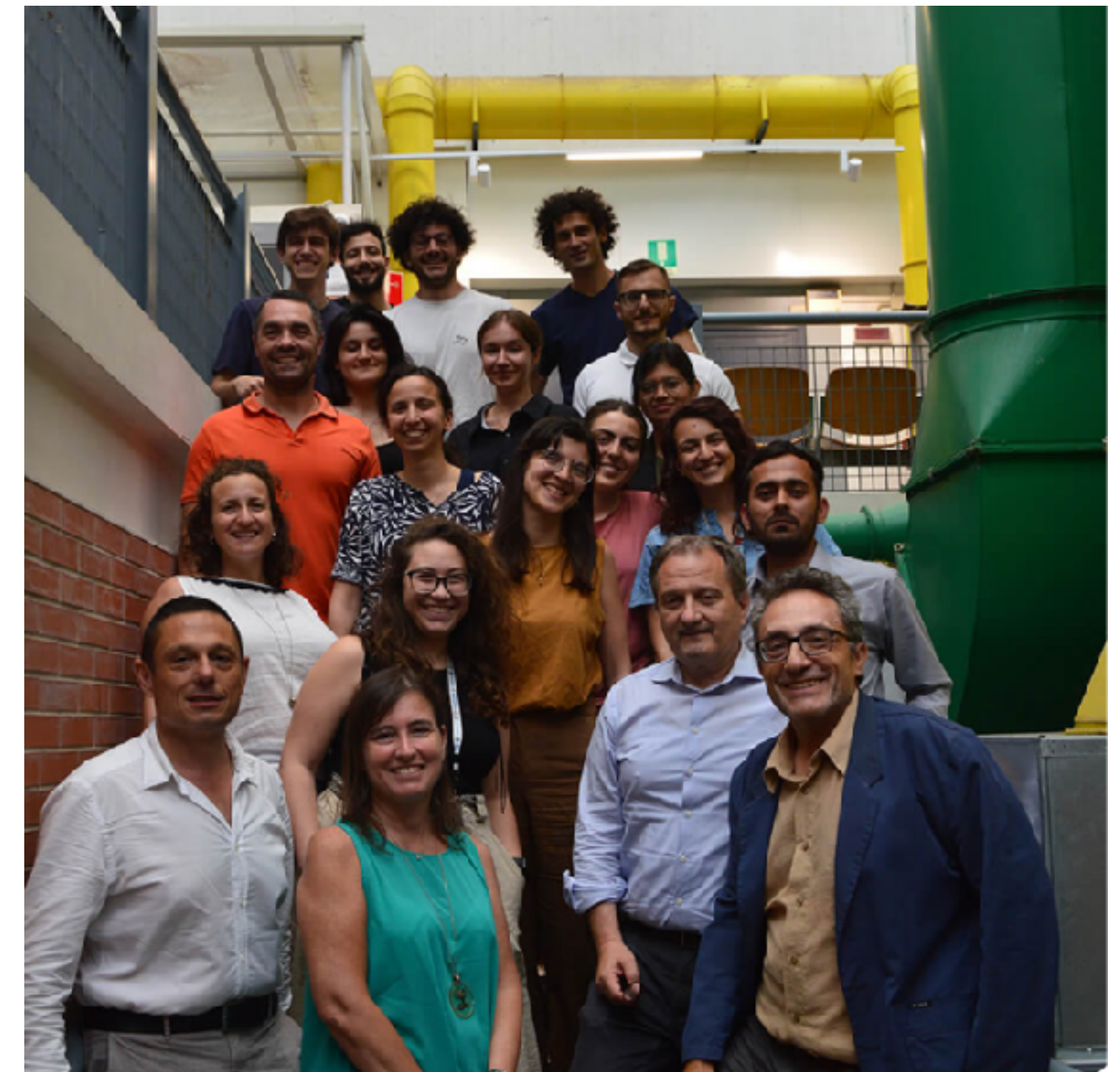


Prof. Tafuri at the unveiling of the first Italian public UniNA

# The collaboration - almost 2 years in the making

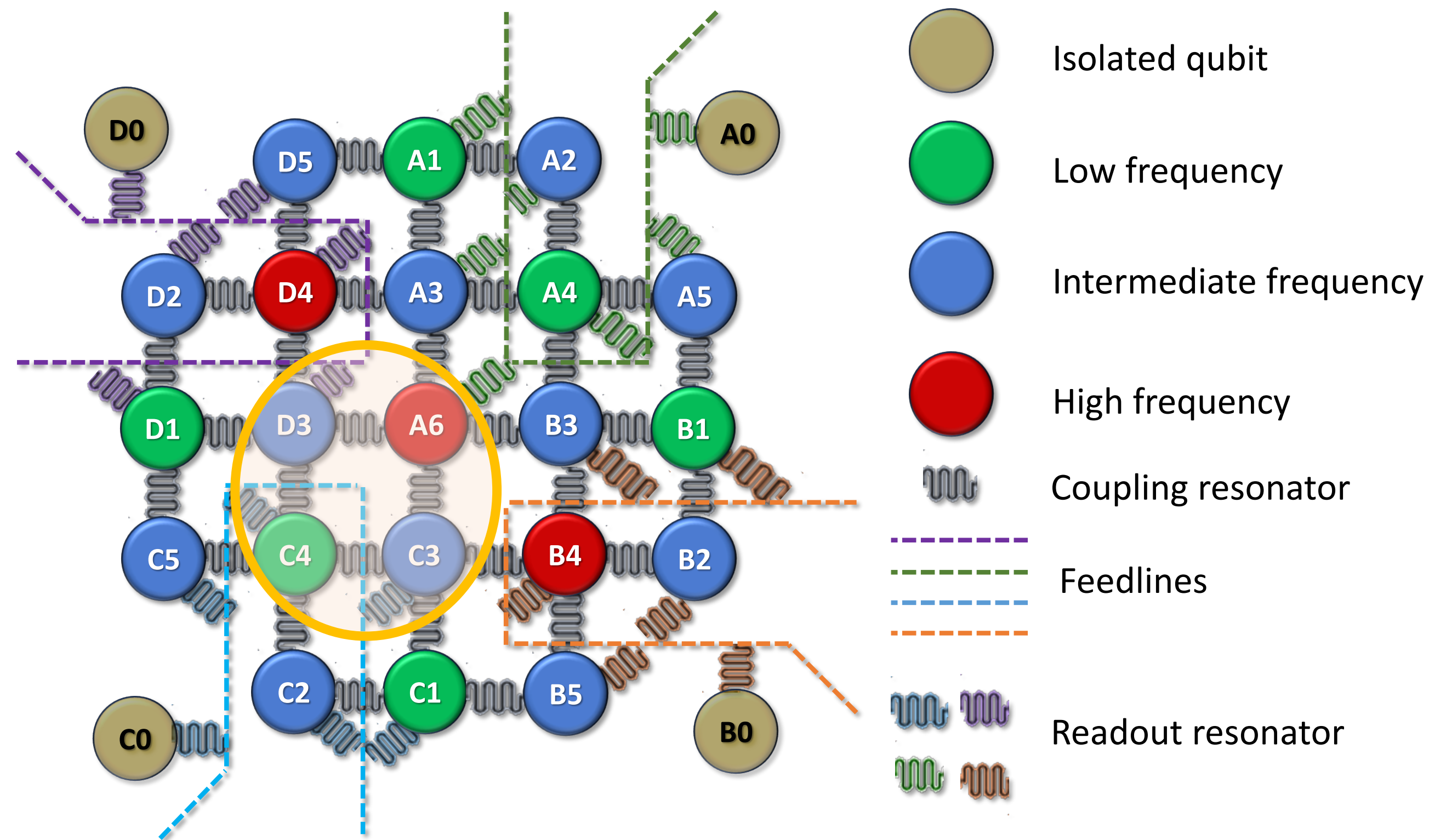


Theory group of Prof. Hamma

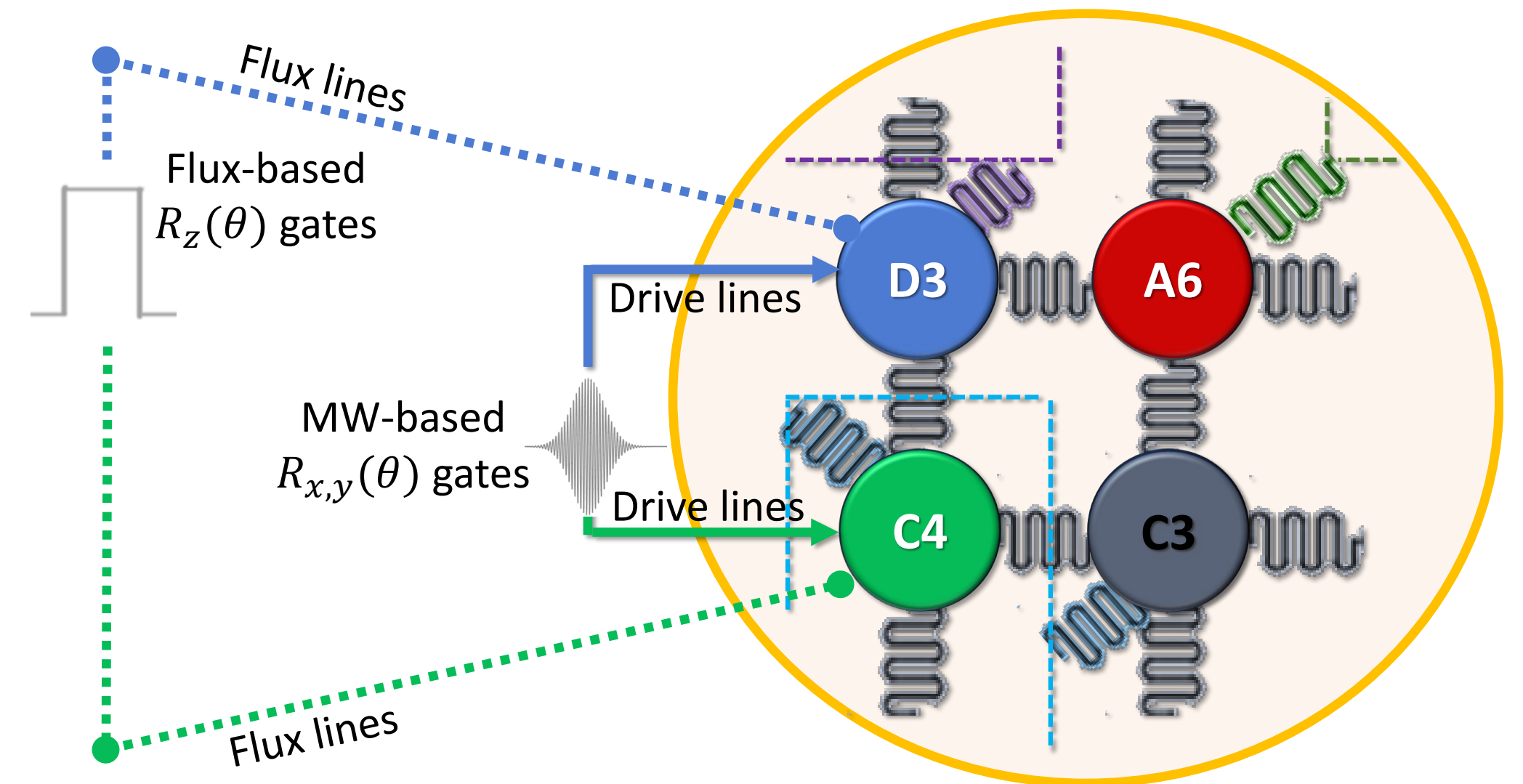


Experimental group of Prof. Tafuri

# The quantum processing unit (QPU) - a Noisy Intermediate Scale Quantum (NISQ) chip



Superconducting Quantum Processing unit. Schematics of the processor. In yellow, isolated fixed-frequency qubits; in red, blue and green the high-, intermediate- and low-frequency qubits, respectively.

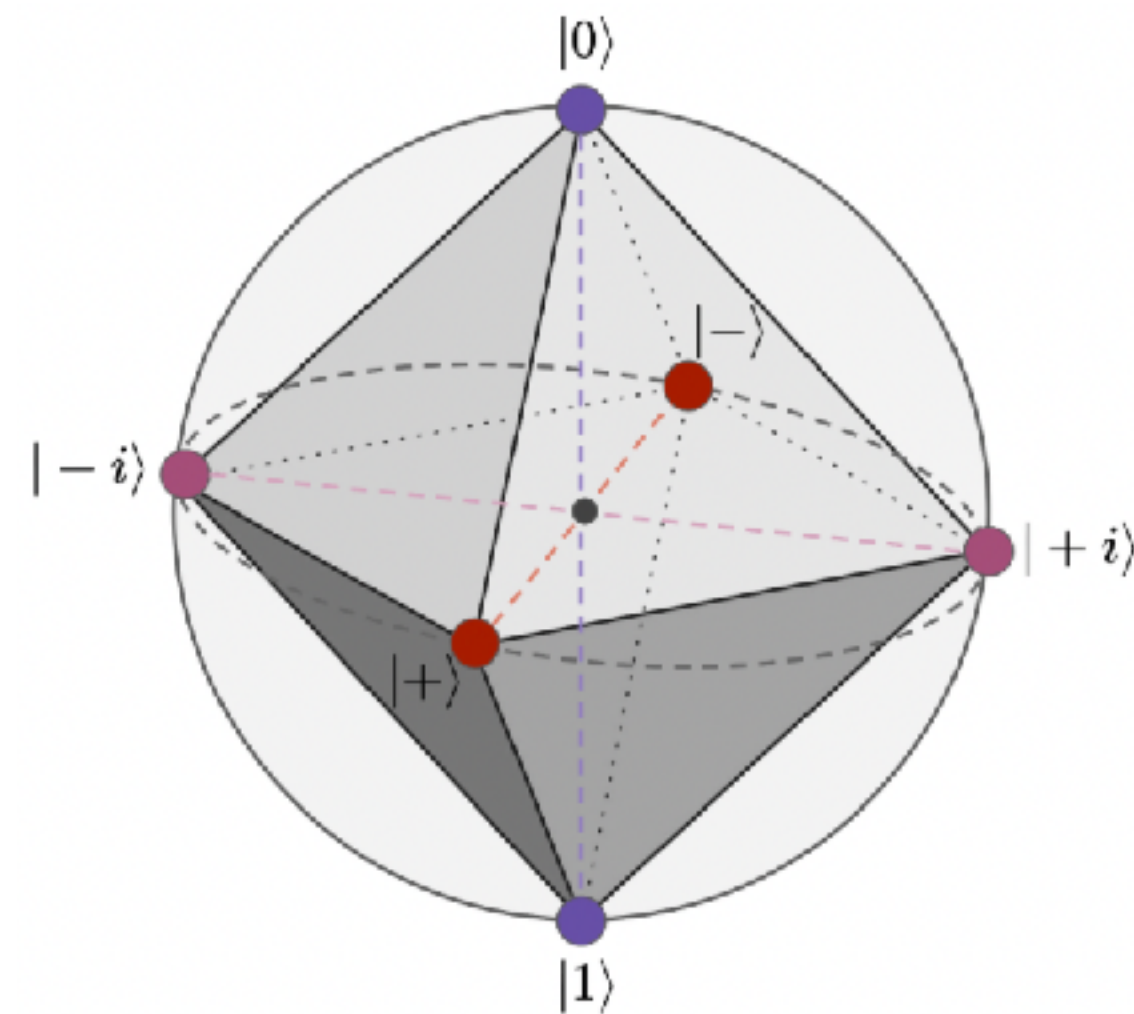


Focus on the investigated pair (D3-C4), coupled with a high-frequency qubit (A6), parked at its sweet spot and a low-frequency qubit (C3) at the anti-sweet spot (in black).

60+ qubit processor coming soon, but also for custom architecture developed at UniNa → for more details ask the experimentalist

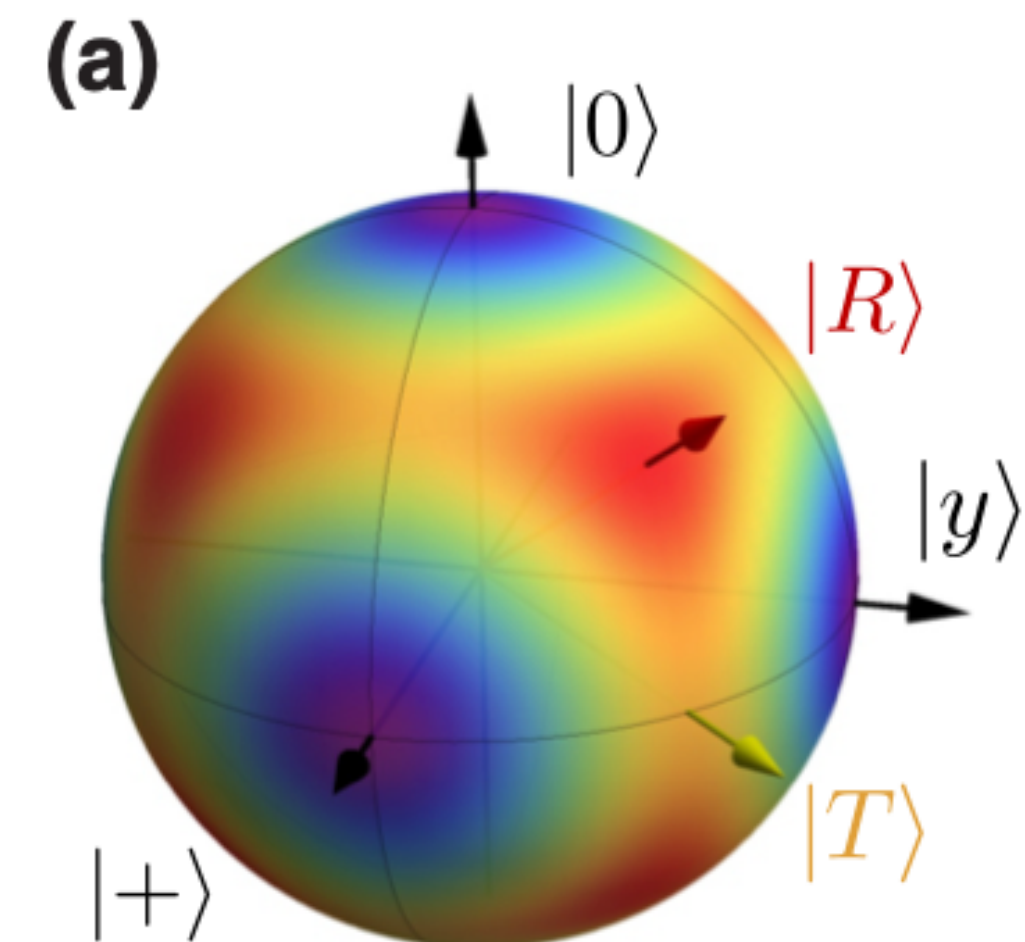
# Let's talk about stabilizer states on the single qubit example

only exploring small patches of the full Hilbert space  
—> 6 pure stabilizer state in case of 1 qubit



The octahedron in the Bloch sphere defines the states accessible via single-qubit Clifford gates (From [https://pennylane.ai/qml/demos/tutorial\\_clifford\\_circuit\\_simulations](https://pennylane.ai/qml/demos/tutorial_clifford_circuit_simulations)).

Non-Clifford gates allows for the exploration of the full Hilbert space

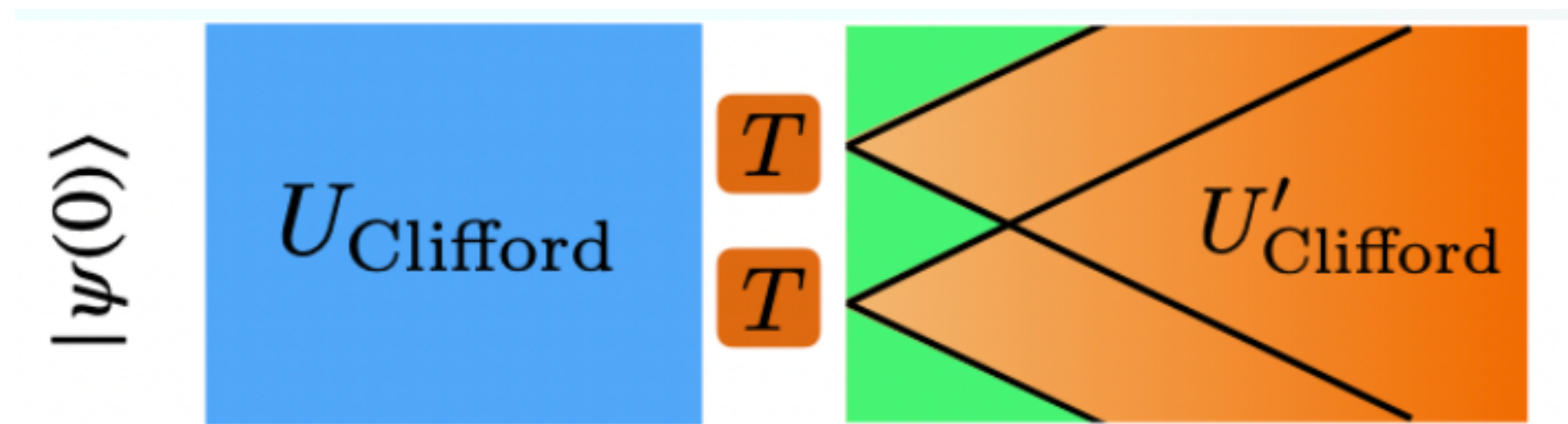


From PRX QUANTUM 4, 010301 (2023)

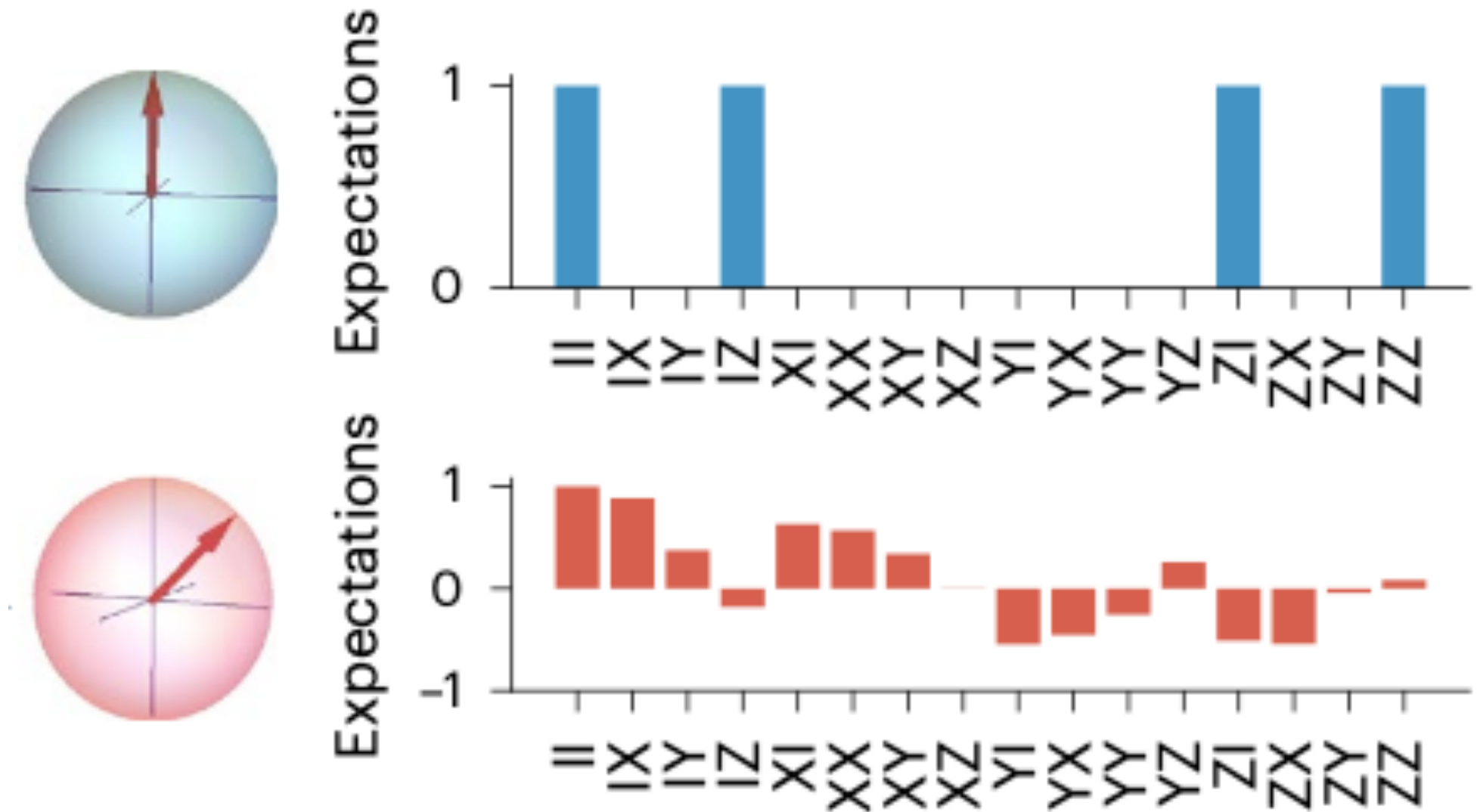


Everything beyond these points—>**non-stabilizerness** or “**magic**”

# The effects of T-gate doping



From SciPost Phys. 9, 087 (2020)



From Niroula, P., White, C.D., Wang, Q. *et al.* Phase transition in magic with random quantum circuits. *Nat. Phys.* 20, 1786–1792 (2024)

**Stabilizer Entropy** as a quantity to capture the the departure from stabilizer states  
(zero for stabilizer states)

L. Leone, S.F.E. Oliviero, A. Hamma PRL **128**, 050402(2022)

$$M_{\alpha}(\psi) = \frac{1}{1-\alpha} \log_2 \mathcal{P}_{\alpha}(\psi), \quad \mathcal{P}_{\alpha}(\psi) = \frac{1}{d} \sum_{P \in \mathcal{P}_N} |\text{Tr}(P\psi)|^{2\alpha}, \quad \psi = |\psi\rangle\langle\psi| \quad d = 2^N$$

# Stabilizer Entropy is efficiently experimentally measurable in multi-qubit systems

## .... sampling via Randomized Measurements

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Probing Rényi entanglement entropy via randomized measurements

TIFF BRYDGES, ANDREAS ELBEN, PETAR JURCEVIC, BENOÎT VERMERSCH, CHRISTINE MAIER, BEN P. LANYON, PETER ZOLLER, RAINER BLATT, AND CHRISTIAN F. ROOS

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**A**

Figure 1A shows a Bloch sphere with a distribution of points. Below it is a histogram of measurement outcomes  $\langle \sigma_z \rangle_u$  ranging from -1.0 to 1.0. The histogram shows two distributions: a narrow blue one for an initial pure state and a wider red one for a mixed state.

**Fig. 1. Measuring second-order Rényi entropies through randomized measurements.** (A) Single-qubit Bloch sphere. The purity is directly related to the width of the distribution of measurement outcomes after applying random rotations  $U_i$ . Initial pure state (blue) and mixed state (red) cases are shown. See text. (B) Generalization to multiple qubits: Measuring  $N_A$ -qubit

**Randomized measurement**

The diagram shows a quantum state with  $N$  qubits entering a box labeled "Randomized measurement". Inside, each qubit line passes through a unitary  $U_i$  (for  $i=1, 2, 3, \dots, N$ ) and then a measurement gate (represented by a meter symbol with a diagonal line). The output is a set of classical bits.

+classical post-processing

personally reminds me of Monte-Carlo sampling

Four square plots labeled "Monte Carlo  $\pi$ " showing the distribution of points. The first plot is empty (white circle on blue background). The second plot has sparse points. The third plot has more points. The fourth plot is densely packed with points, illustrating the convergence of Monte Carlo sampling to the area of the circle.

# What is Non-Local magic?

- Non-Local magic appears when magic gets **spread around** in a state with entanglement: [PRX Quantum 6, 040375 \(2025\)](#)
- Intrinsically related to entanglement.
- Protocol: preparing non-local states and trying to **erase it** locally.
- **NL magic has not yet never been measured!**

$$M_2^{\text{NL}}(|\psi\rangle) = \min_{U_A, U_B} M_2(U_A \otimes U_B |\psi\rangle)$$



Artistic visualization of entanglement  
between two spins/qubits

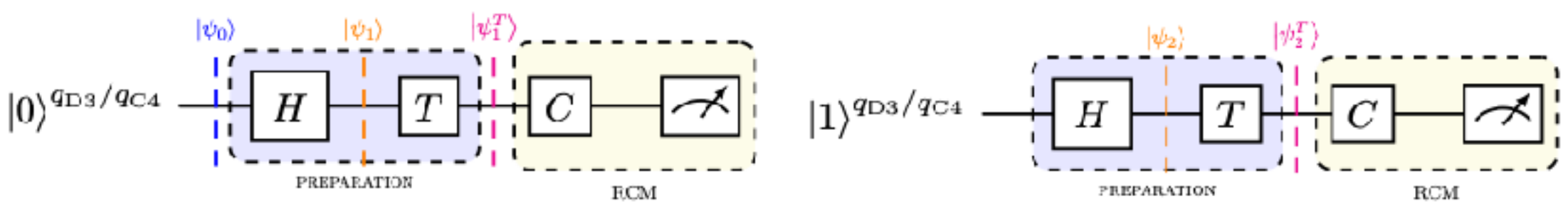
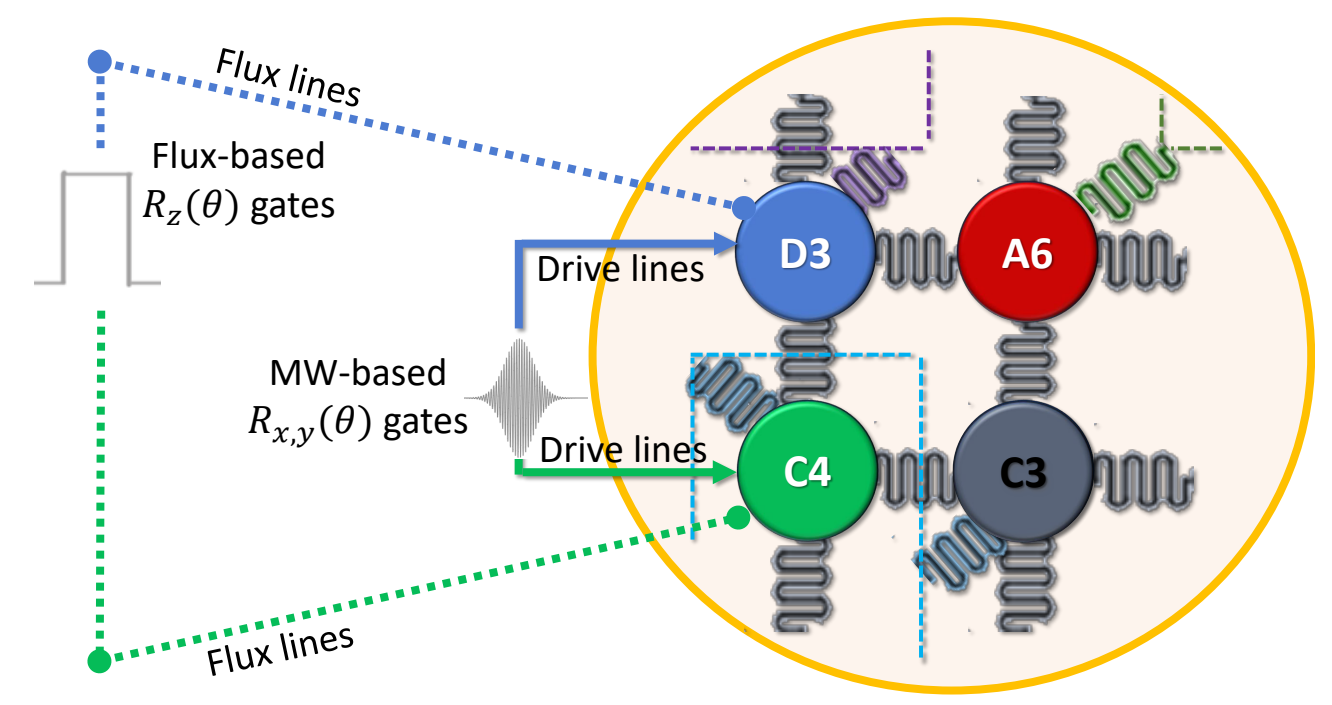
NL magic has not yet been experimentally measured!



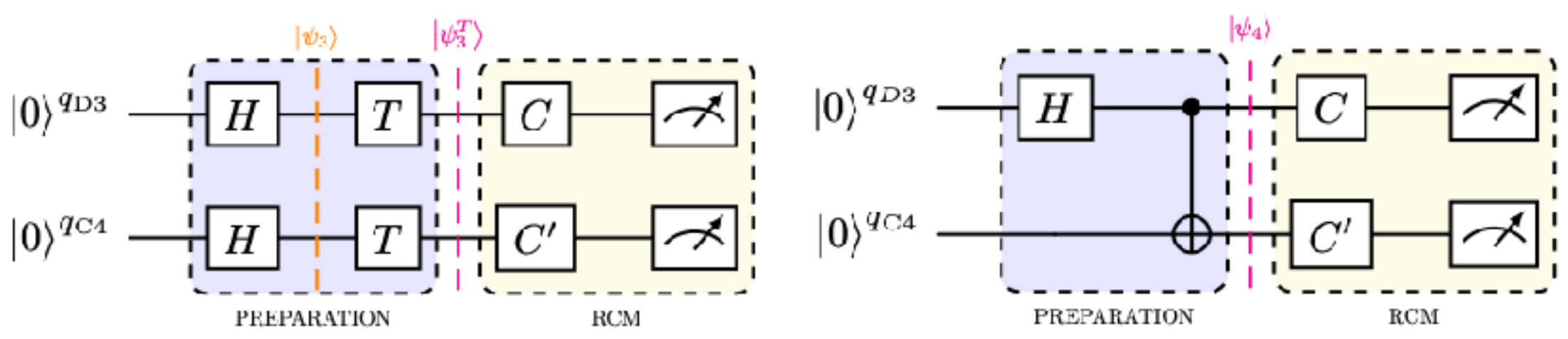
So we do it!

|               | Class 1         | Class 2                  | Class 3              |
|---------------|-----------------|--------------------------|----------------------|
| Notation      | Local Magic(LM) | Local+Non-Local Magic(M) | Non-Local Magic(NLM) |
| Local         | ✓               | ✓                        | ✗                    |
| Non-Local     | ✗               | ✓                        | ✓                    |
| Local Erasure | Complete        | Partial                  | Null                 |

# Benchmarks and identification of noise channels



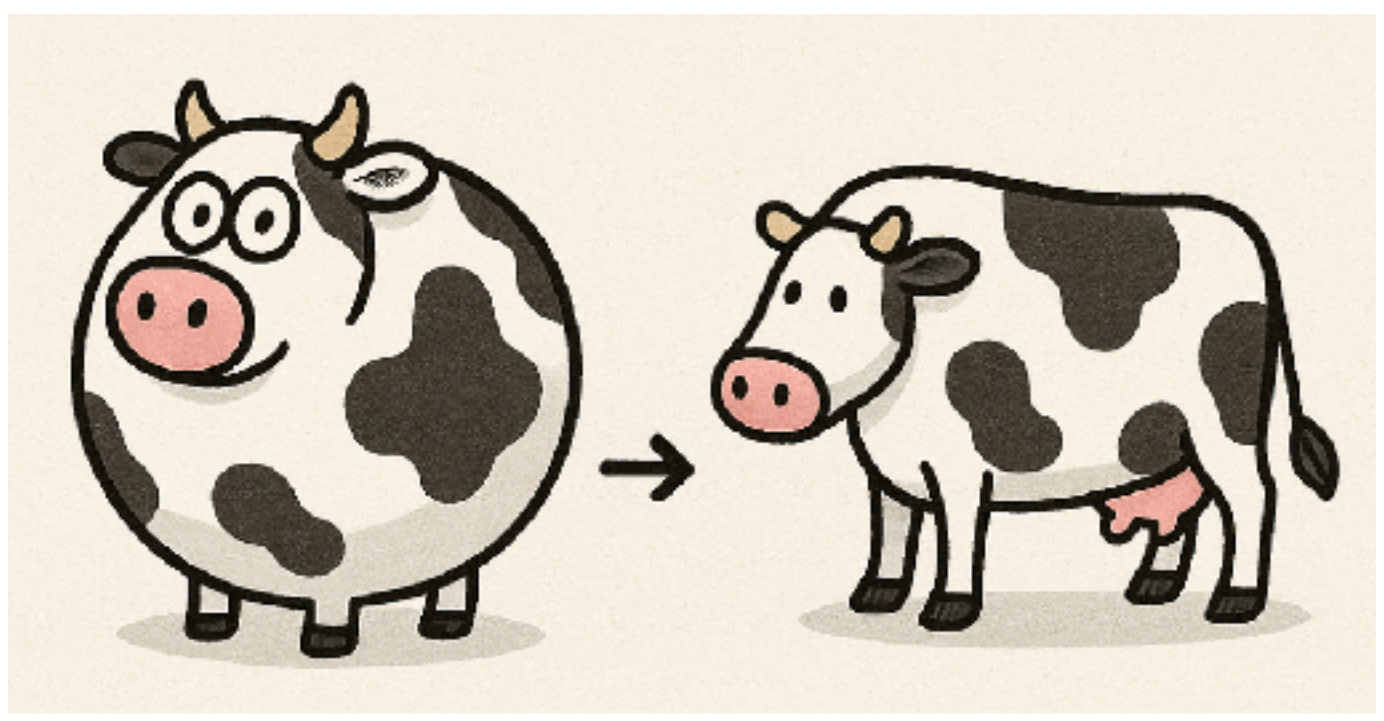
(a) Single-qubit state initially in the  $|0\rangle$  state. (b) Single-qubit state initially in the  $|1\rangle$  state.



(c) Separable two-qubit states. (d) Bell state.

|     | $F_{RO}(\%)$ | $F_{RB}(\%)$       | $T_2^{\text{Echo}}(\mu s)$ | $T_2^*(\mu s)$   | $T_1(\mu s)$      | $M_2^{\text{exp}}(\Psi^{M_2^{1/2}=0})$ | $M_2^{\text{exp}}(\Psi^{M_2^{1/2}\neq 0})$ |
|-----|--------------|--------------------|----------------------------|------------------|-------------------|----------------------------------------|--------------------------------------------|
| (a) |              |                    |                            |                  |                   |                                        |                                            |
| D3  | $94 \pm 1$   | $99.75 \pm 0.01$   | $31.1 \pm 0.7$             | $25 \pm 6$       | $23.3 \pm 0.1$    | $ \Psi_0\rangle$                       |                                            |
|     |              |                    |                            |                  |                   | $0.1 \pm 0.1$                          |                                            |
|     |              |                    |                            |                  |                   | $ \Psi_1\rangle$                       | $ \Psi_1^T\rangle$                         |
|     |              |                    |                            |                  |                   | $0.1 \pm 0.1$                          | $0.5 \pm 0.1$                              |
|     | $96 \pm 1$   | $99.910 \pm 0.002$ | $39.9 \pm 0.7$             | $33 \pm 6$       | $30.0 \pm 0.1$    | $ \Psi_1\rangle$                       | $ \Psi_1^T\rangle$                         |
|     |              |                    |                            |                  |                   | $0.1 \pm 0.1$                          | $0.4 \pm 0.1$                              |
|     |              |                    |                            |                  |                   | $ \Psi_2\rangle$                       | $ \Psi_2^T\rangle$                         |
|     |              |                    |                            |                  |                   | $0.1 \pm 0.1$                          | $0.4 \pm 0.1$                              |
| C4  | $95 \pm 1$   | $99.925 \pm 0.003$ | $38.8 \pm 1.1$             | $24 \pm 2$       | $36.2 \pm 0.1$    | $ \Psi_1\rangle$                       | $ \Psi_1^T\rangle$                         |
|     |              |                    |                            |                  |                   | $0.1 \pm 0.1$                          | $0.4 \pm 0.1$                              |
|     |              |                    |                            |                  |                   | $ \Psi_2\rangle$                       | $ \Psi_2^T\rangle$                         |
|     |              |                    |                            |                  |                   | $0.1 \pm 0.1$                          | $0.4 \pm 0.1$                              |
| (b) |              |                    |                            |                  |                   |                                        |                                            |
|     | $F_{RO}(\%)$ | $F_{RB}(\%)$       | $F_{RB}^{CZ}(\%)$          | $Cross^{MW}(\%)$ | $Cross^{ZZ}(kHz)$ |                                        |                                            |
| D3  | $83 \pm 1$   | $99.746 \pm 0.014$ | $99 \pm 2$                 | $\sim 0.12$      | $\sim 99$         | $ \Psi_3\rangle$                       | $ \Psi_3^T\rangle$                         |
|     |              |                    |                            |                  |                   | $0.1 \pm 0.1$                          | $0.8 \pm 0.1$                              |
| C4  | $91 \pm 1$   | $99.905 \pm 0.003$ |                            | $\sim 1.57$      | $\sim 100$        | $ \Psi_3\rangle$                       | $ \Psi_3^T\rangle$                         |
|     |              |                    |                            |                  |                   | $0.1 \pm 0.1$                          | $0.8 \pm 0.1$                              |
| (c) |              |                    |                            |                  |                   |                                        |                                            |
|     |              |                    |                            |                  |                   | $ \Psi_4\rangle$                       |                                            |
| D3  | $92 \pm 1$   |                    |                            |                  |                   | $0.2 \pm 0.1$                          |                                            |
| C4  |              |                    |                            |                  |                   |                                        |                                            |

**Table 1: Experimental magic measured for the quantum states in Fig. 3.** Systematic investigation of the experimental magic for single- and two-qubit quantum states, involving qubits D3 and C4, as a function of: the readout fidelity  $F_{RO}$ , the average single-qubit gates fidelity measured by Randomized Benchmarking,  $F_{RB}$ , both in the isolated case (a) and simultaneously (b), statistical relaxation, Ramsey and Echo coherence times ( $T_1$ ,  $T_2^*$  and  $T_2^{Echo}$ , respectively). For the two-qubit circuits, we have also reported an estimation of the drive and ZZ crosstalk on the investigated pair [55–58]. A discussion on the error estimation is reported in App. B.



# Our job as theoreticians in this case

**...identify and understand the noise channels from a big set of preliminary/benchmark data**

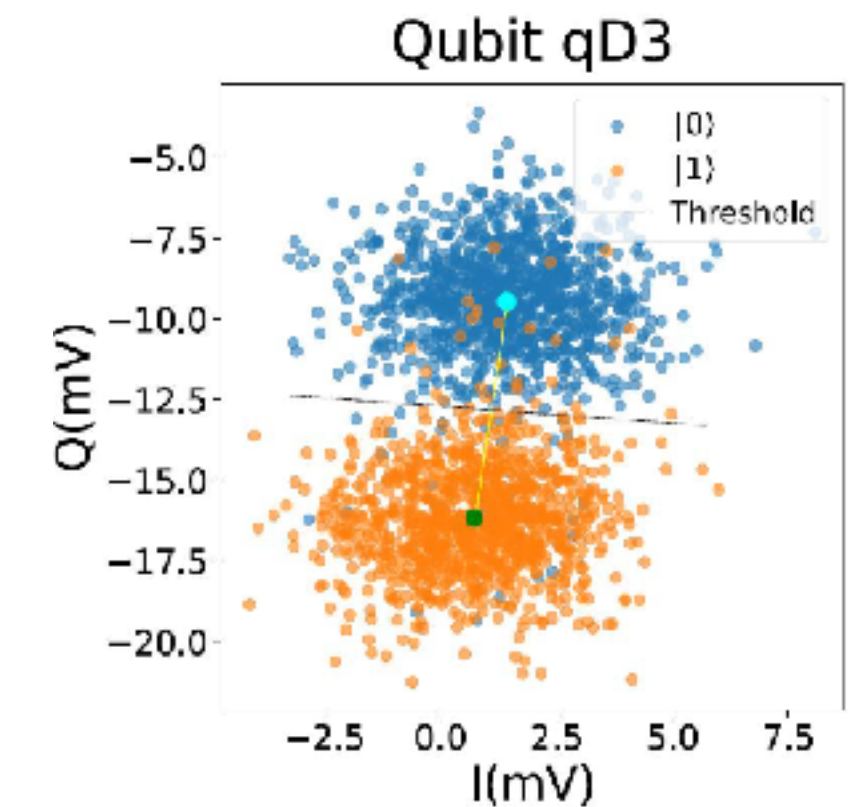
## 1.2 Strategies to counter errors

1. **Do nothing and hope for the best.** We only run short circuits before our computation is overwhelmed by the noise. This regime is called the *Noisy Intermediate-Scale Quantum (NISQ)* regime [16].

Error Mitigation and Error Correction Lecture Notes, Philippe Faist

Readout errors occur at the point of measurement, where, in the act of measurement, a bit string element is randomly flipped as

$$0 \rightarrow 1 \quad \text{or} \quad 1 \rightarrow 0.$$



## 1. Measurement Readout Error

## 2. CNOT Gate Error (depolarizing noise)

$$\text{CNOT}_{1 \rightarrow 2} = \begin{array}{c} \bullet \\ | \\ \oplus \end{array} = \begin{array}{c} \bullet \\ | \\ \boxed{H} \text{---} \boxed{Z} \text{---} \boxed{H} \end{array} = \begin{array}{c} \bullet \\ | \\ \boxed{H} \text{---} \bullet \text{---} \boxed{H} \end{array}$$

$$\mathcal{E}(\rho) = \frac{pI}{2} + (1-p)\rho. \quad (8.100)$$

The effect of the depolarizing channel on the Bloch sphere is illustrated in Figure 8.11.

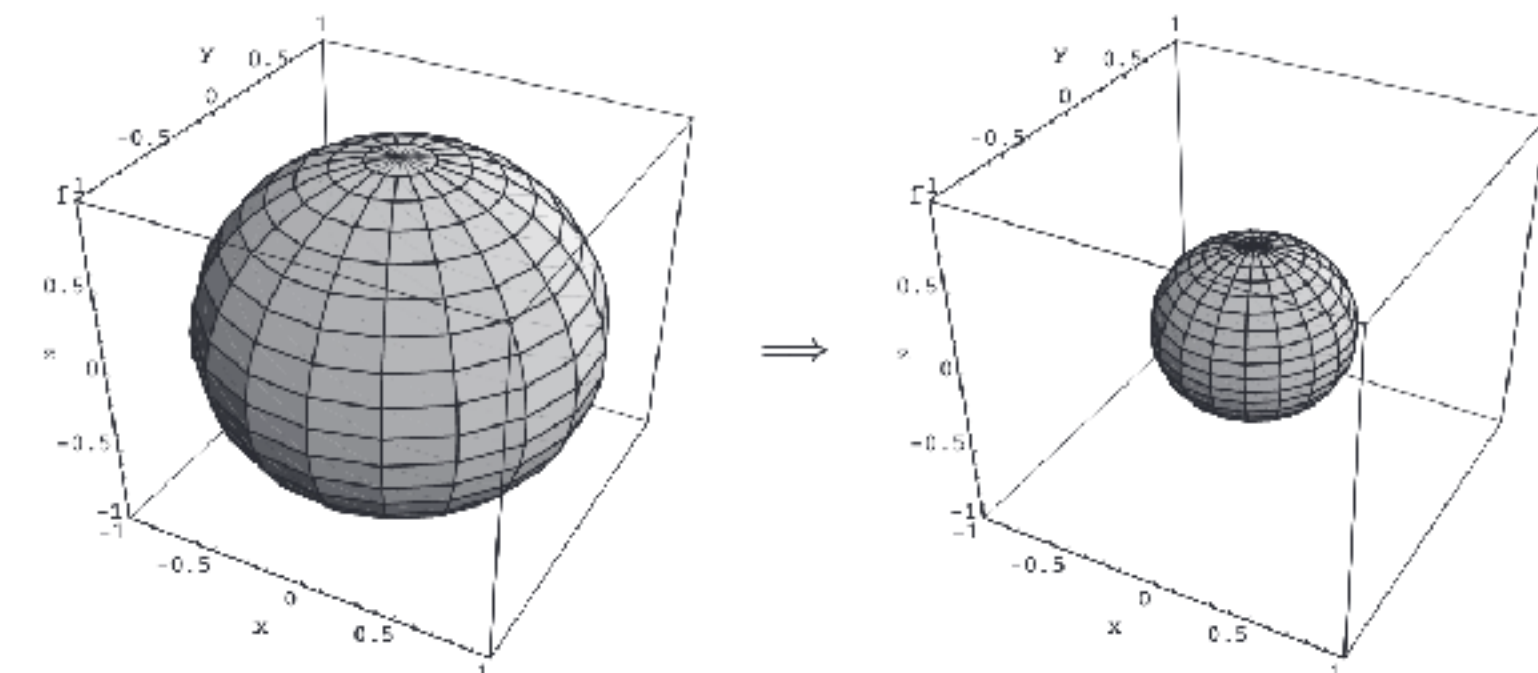
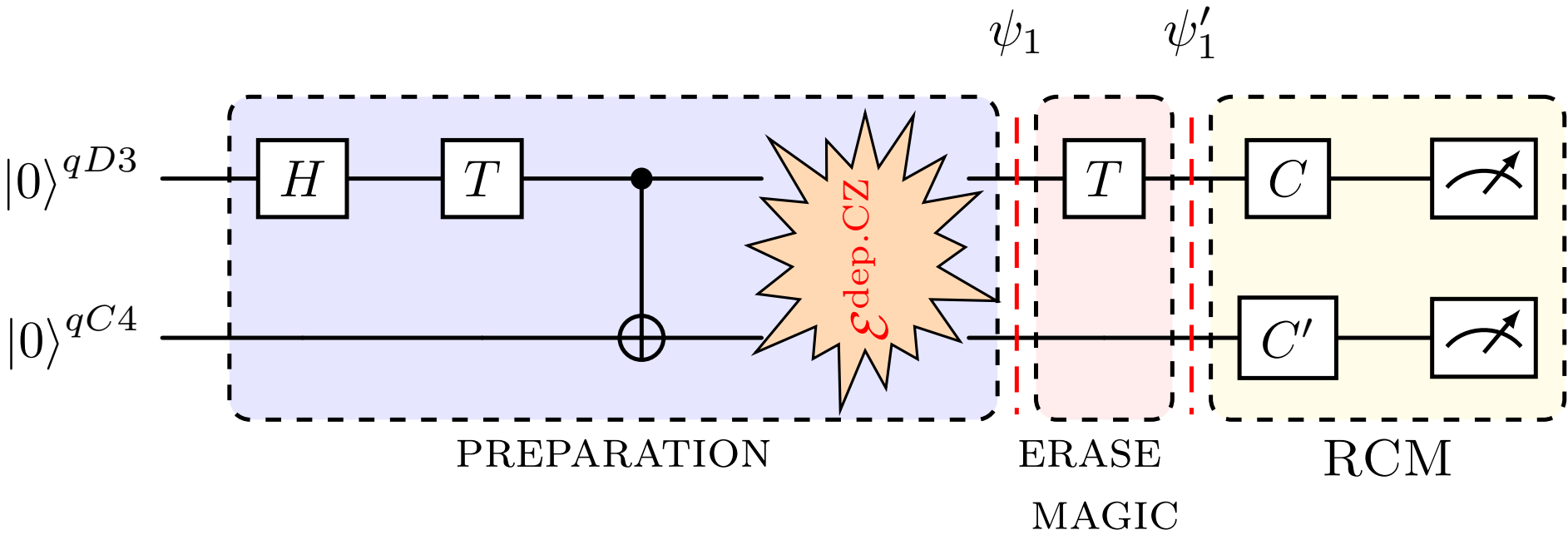


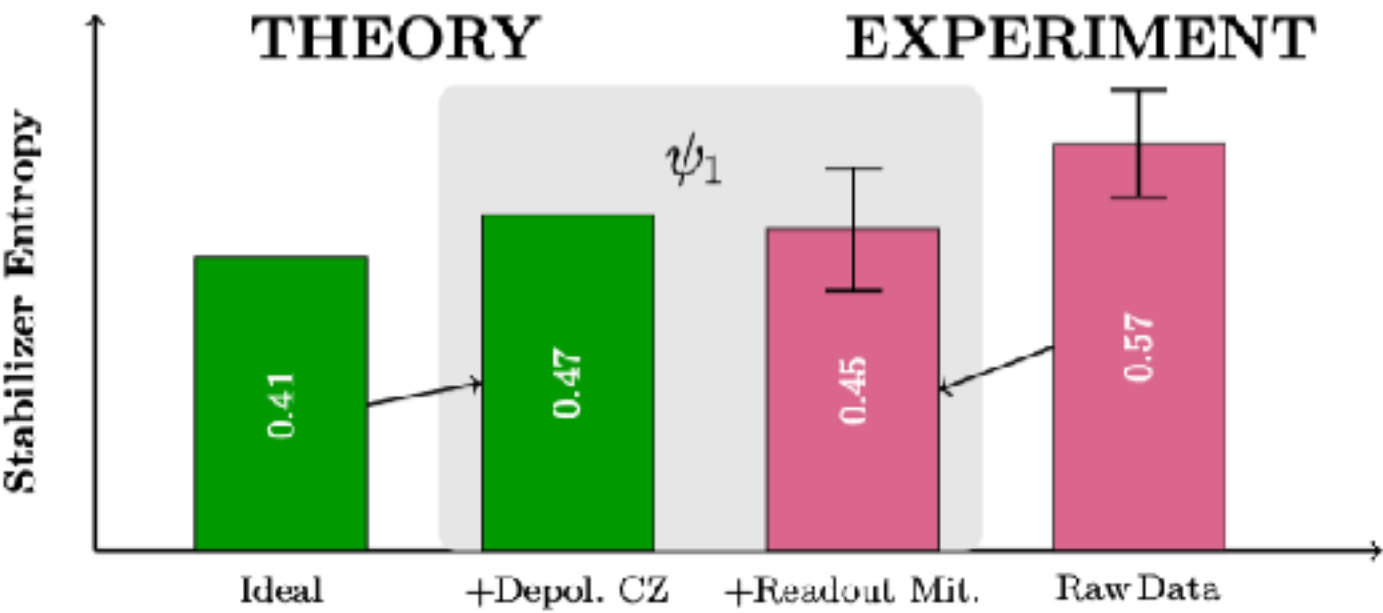
Figure 8.11. The effect of the depolarizing channel on the Bloch sphere, for  $p = 0.5$ . Note how the entire sphere

# Class 1: Local Magic (L)



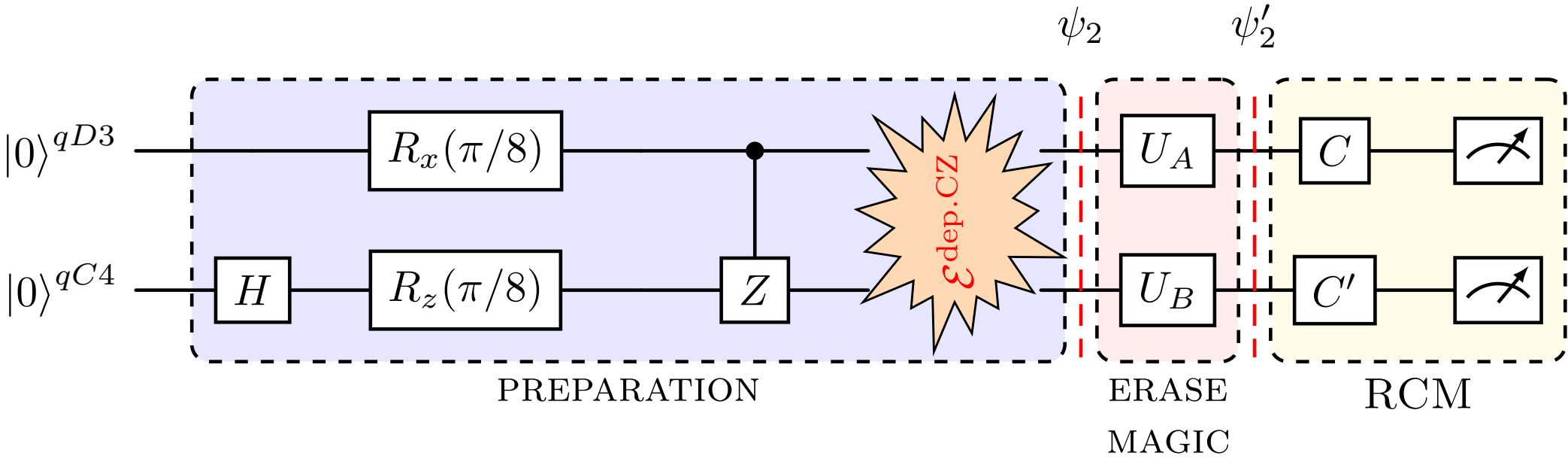
| Circuit       | State     | Th. Purity | Exp. Purity     | Th. Magic | Exp. Magic      | NL-Magic (from RDM purity) |
|---------------|-----------|------------|-----------------|-----------|-----------------|----------------------------|
| Preparation   | $\psi_1$  | 0.92       | $1.02 \pm 0.04$ | 0.47      | $0.45 \pm 0.06$ |                            |
| Prep.+Erasure | $\psi'_1$ | 0.92       | $1.05 \pm 0.06$ | 0.11      | $0.08 \pm 0.08$ | $0.1176 \pm 0.0007$        |

**Table 2: Local Magic (LM) state results.** Comparison of Purity and Magic for states  $\psi_1$  (before) and  $\psi'_1$  (after) the erasure, as well as the value of non-local magic obtained from subsystem purity. The theoretical predictions for purity and magic are calculated by considering the CZ depolarizing error and compared with the experimental results after readout error mitigation. Details on the errors are reported in *Methods*.



Alternative but not as scalable method to evaluate NL magic

# Class 2: Local+Non-Local Magic (M)



$$U_A = R_z(\alpha)R_y(\beta)R_z(\gamma) \text{ , } U_B = R_z(\delta)R_y(\eta)R_z(\phi) \text{ ,}$$

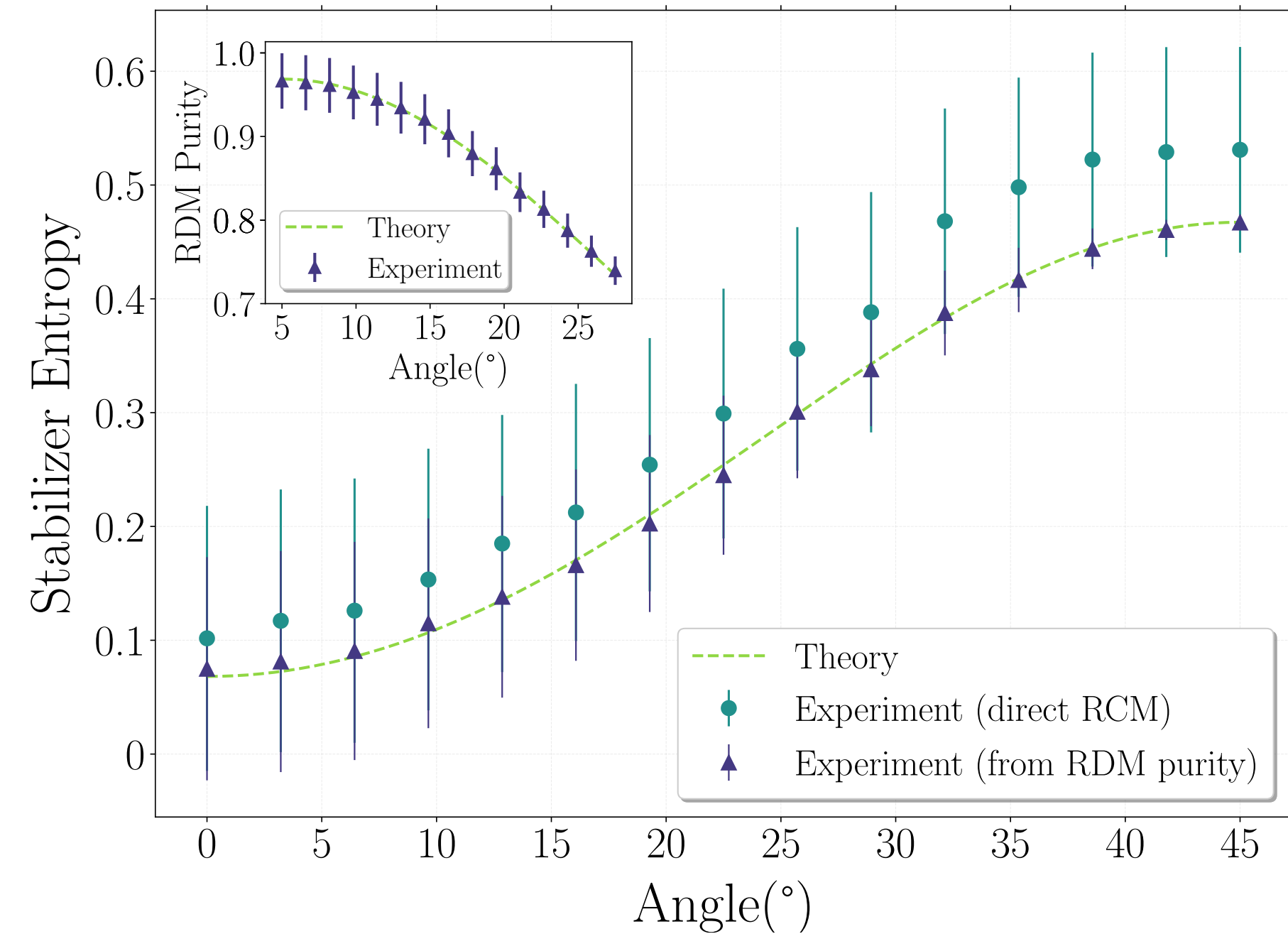
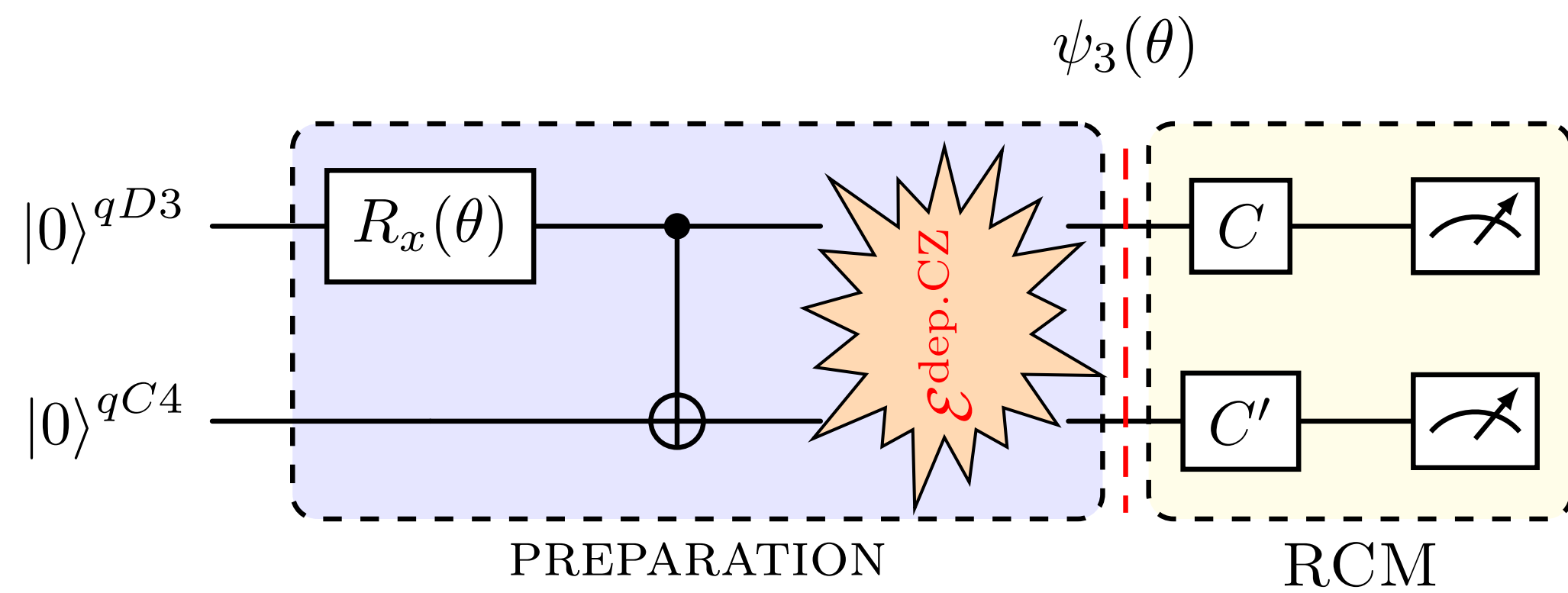
$$\alpha = \beta = \gamma = \delta = \eta = 0, \phi = 67.61^\circ$$

What’s left is non-local magic!

| Circuit       | State     | Th. Purity | Exp. Purity     | Th. Magic | Exp. Magic      | NL-magic (from RDM purity) |
|---------------|-----------|------------|-----------------|-----------|-----------------|----------------------------|
| Preparation   | $\psi_2$  | 0.95       | $0.94 \pm 0.04$ | 0.44      | $0.42 \pm 0.09$ |                            |
| Prep.+Erasure | $\psi'_2$ | 0.95       | $1.00 \pm 0.05$ | 0.3       | $0.3 \pm 0.1$   | $0.18 \pm 0.07$            |

**Table 3: Local + Non-Local (M) magic quantum state results.** Comparison of Purity and Magic for states  $\psi_2$  (before) and  $\psi'_2$  (after) the erasure protocol, as well as the value of non-local magic obtained from subsystem purity. The theoretical predictions for purity and magic are calculated by considering the CZ depolarizing error and compared with the experimental results after readout error mitigation. Details on the errors are reported in *Methods*.

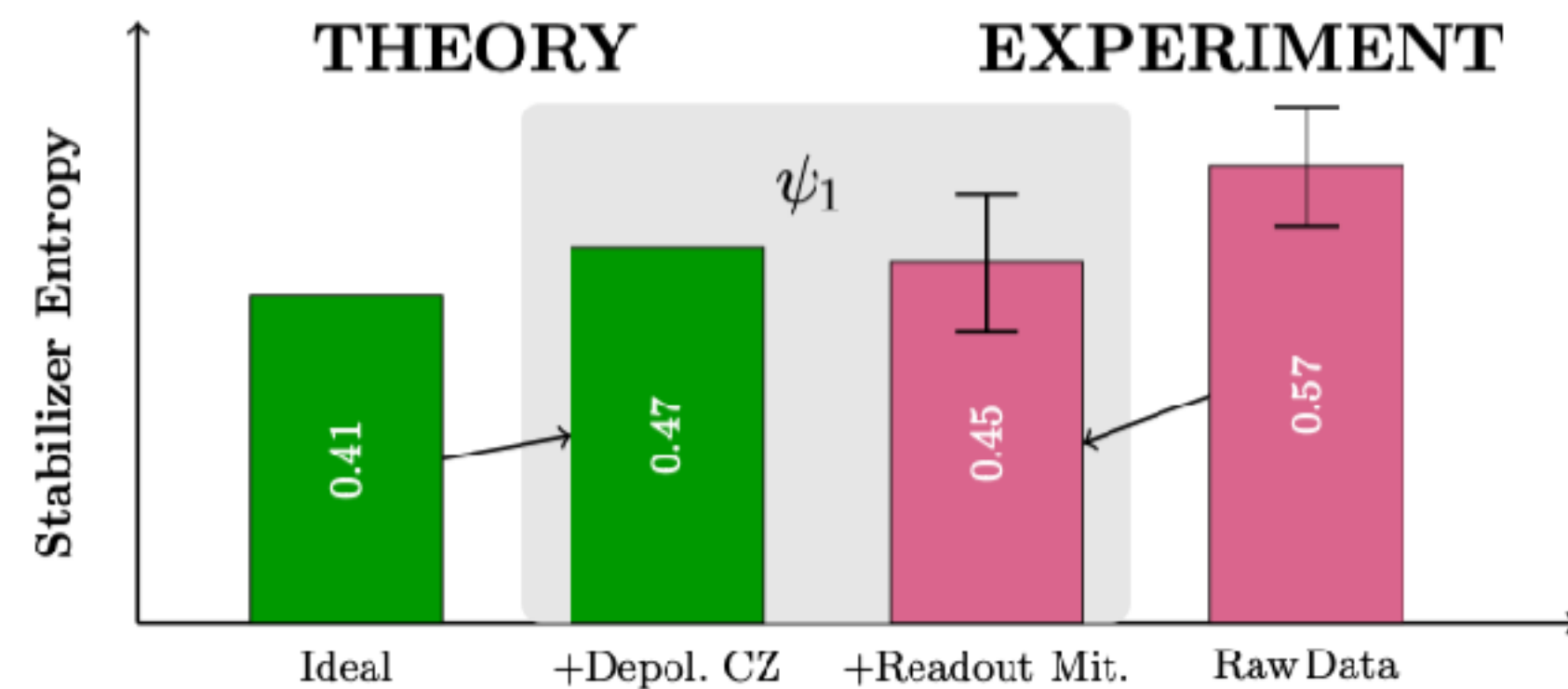
# Class 3: Non-Local Magic (NLM)



All together —> **smoking gun** for experimental non-local magic in a superconducting QPU

# Conclusions and Outlook

- Indeed a love story between experimentalist and theorists with a happy ending 💙
- First experimental demonstration of non-local magic using in-depth knowledge of QPU noise channels
- Error Mitigation applicable to all other future experiments of the UniNA QPUs
- RCM approach extendable large qubit numbers and other observables
- Non-Local magic in relation to AdS/CFT (arXiv:2403.07056)



**The story of bringing  
experiment and theory  
together**